

Delegated Contracting, Information, and Internal Control

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Economics Letters, forthcoming - accepted for publication 6/30/2008

Abstract

This paper studies a particular form of internal control that a (top) principal can use when delegating the offer of a Baron-Myerson style subcontract to an intermediary. It is shown that the information structure gives the principal some leeway to reduce the loss of control when there is no communication between the highest and lowest tier.

JEL classification: D23, D73, D82, L14, L50.

Keywords: Delegated Contracting, Value of Communication, Vertical Hierarchies, Internal Control, Managerial Accounting, Agency Costs.

1 Introduction

What is the value of communication between the top two layers of a vertical hierarchy? Can the principal improve sequential contracting through structuring communication with an intermediate player (manager, prime contractor) to whom he delegates the offer of a partially verifiable sub-contract? This question is of particular interest for the analysis of delegated contracting, a scheme in which the principal cannot interact with the productive agent at the lowest layer, but relies on designing output targets and budgets together with some possible form of internal control.

Delegated contracting schemes have been studied within the larger literature on centralized versus decentralized contracting, to explain the particular loss of control arising in public bureaucracies and large firms. Melumad, Mookherjee and Reichelstein (1992) have shown that three-tier hierarchies do not come with additional costs if top-down contracting permits the principal to tax away any rent of the intermediary *ex ante*. In turn, McAfee and McMillan's (1995) seminal paper shows that even if the intermediary¹ has no private information, her being protected by limited liability leads to a double-marginalization of rents, a typical control loss affecting vertical relationships. In the view of Melumad, Mookherjee and Reichelstein (1997), delegated contracting still offers the potential advantage of giving the intermediary full information about the agent's type when designing the subcontract.

Faure-Grimaud and Martimort (2001) (hereafter, FGM) have redefined delegated contracting to include the task of forwarding a Baron-Myerson (1982)² style contract to the agent. Information, rents, and communication now become intertwined in a particular way: the intermediary is hired because of her ability to costlessly filter out an unwanted third type of agent *before* offering a BM contract to the two remaining types that she still

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¹As common in this literature, I use female pronouns for the intermediary and male for the principal.

²BM hereafter.

cannot distinguish. This gives the intermediary some option to use her ex-ante information to her advantage, increasing the principal’s risk of being left without production.³

To further close the lacuna on delegated contracting, this paper explores the principal’s options to make use of existing and accessible information in order to limit the discretion given to players located at lower tiers. In FGM, delegated contracting is carried out in an output- and message-contingent grand contract based on information prior to (sub-) contracting. Using a state-contingent analysis, I explore the underlying problem of partial verifiability of the subcontract that follows from the noncontractibility of the intermediary’s offer. Melumad and Reichelstein (1987 and 1989) have already shown that message-dependent contracts with communication may reveal an advantage over BM contracts. This paper follows their original idea, although the intermediary is hired to offer only a BM contract. Baron and Besanko (1992) have studied a similar delegated contract, arguing that under the presence of informational asymmetries it is the organizational structure, the locus of information, and the nature of communication that call for the “designer’s choice of the scope of control.”⁴ While their ‘Hierarchy III’ comes close to the setting studied in this paper, the contracting scheme used here needs to cope with the problem of partial verifiability of the intermediary’s action.

This paper’s contribution is twofold. First, it provides an extended analysis of how communication and information flow can be structured together with the choice of contracting. In FGM, communication between principal and intermediary is possible, but their analysis does not focus on how decision making can be improved when the principal has a choice between centralized and delegated contracting. Second, the paper shows that a simple form of control may improve delegated contracting. This form of control permits some discussion of related benchmarks. Monitoring financial information together with the form of budgeting and output targets is already used in FGM. It is thus straightforward to extend managerial accounting toward internal control performed through an examination of the written document the intermediary must be knowledgeable of. This form of control differs from standard forms of auditing where the principal aims at detecting the *type* of a productive agent.⁵ This paper thus offers a rationale for particular forms of control typically used in vertical hierarchies which no access to the productive agent.

2 The model

2.1 General setup and the intermediary’s rents in the original FGM setting

This section offers a generalized notation to extend the stylized contractual scheme of FGMs three player model with a (top) principal, an intermediary, and a productive agent (firm). The productive agent can be of three types: $\underline{\theta}$, $\hat{\theta}$ and $\bar{\theta}$, with $\Delta\theta \equiv \hat{\theta} - \underline{\theta}$. Type $\bar{\theta}$ has excessive marginal cost, and should therefore never be offered a contract according to the principal’s will. To avoid having the $\bar{\theta}$ -type in the regime, the principal hires the intermediary who can costlessly observe type $\bar{\theta}$ with a known precision p . When $\bar{\theta}$ is not detected, the agent’s type can still be $\underline{\theta}$ (with a probability ν) or $\hat{\theta}$ (with a probability $1 - \nu$) at the subcontracting stage. The resulting priors are $(1 - p)\nu$, $(1 - p)(1 - \nu)$, and p , respectively. The principal maximizes his expected net surplus $S(q) - s$, which includes the budget s paid to the intermediary to forward production transfers t to the productive agent. Transfers are designed to permit the offer of a BM style contract to the agent that includes the virtual costs $\Delta\theta\hat{q}$, which induces production by both types $\underline{\theta}$ and $\hat{\theta}$. $S(q)$ denotes the principal’s gross surplus, with $S' > 0$, and $S'' < 0$.

³For a detailed overview and classification of contributions see Mookherjee (2005, p. 11) who compares the two settings of McAfee and McMillan (1995) and FGM from a risk perspective.

⁴Baron and Besanko (1992, p. 239). FGM’s setting can also be seen as a borderline case of collusion in which no communication is needed between intermediary and agent, her always reaping the benefits of collusion when successful. Second, the intermediary’s contract offer always involves the informed principal problem; revealing a piece of information that the intermediary possesses. Since this choice is binary and involving access to the agent, this chain of arguments cannot be followed here. For a treatment see Laffont and Martimort (1998).

⁵This type is always known ex post when a subcontract was signed.

In FGM's model, the agent's rents follow the basic screening setup with two players, making it possible to express the intermediary's rents as $\underline{v} = \underline{s} - \underline{\theta}q$ and $\hat{v} = \hat{s} - \hat{\theta}\hat{q}$ following the use of budgeting. These rents also reflect the surplus of the intermediary-agent coalition. The intermediary furthermore is assumed to be risk averse, with a VNM utility function of $V(s - t) = \frac{1 - e^{-r(s-t)}}{r}$, and r denoting her degree of risk aversion.

In the original FGM setting, delegation proofness is reached by satisfying the following constraints of the intermediary. That is, two downward incentive constraints, namely

$$\underline{v} \geq \hat{v} + \Delta\theta\hat{q} \tag{1}$$

and

$$\nu V(\underline{v} - \Delta\theta\hat{q}) + (1 - \nu)V(\hat{v}) \geq \nu V(\underline{v}) + (1 - \nu)V(s(\bar{q})). \tag{2}$$

make sure that the intermediary prefers to delegate to the $\underline{\theta}$ -type agent, plus delegating BM contract instead of a shutdown contract. Moreover, two (interim) more participation constraints need to be met. These are

$$V(\bar{s}) \geq 0 \tag{3}$$

and

$$\nu V(\underline{v} - \Delta\theta\hat{q}) + (1 - \nu)V(\hat{v}) \geq 0. \tag{4}$$

Both (3) and (4) make sure that the intermediary, in addition to her risk aversion, is always protected by limited liability below zero wealth, whether the sub-contract is accepted or rejected by the agent.

Since the intermediary's contract offer is unobserved, she can require a rent that prevents her from offering a shutdown contract to the agent, which in FGM (2001, p. 80) leads to the following incentive constraint:

$$V(\hat{v}) \geq \nu V(\hat{v} + \Delta\theta\hat{q}). \tag{5}$$

The intermediary can only costlessly detect a $\bar{\theta}$ -type agent. If not observing this type, she could still offer a shutdown contract and reaping the virtual cost $\Delta\theta\hat{q}$ herself. To restore delegation proofness, a rent is paid to satisfy (5).

2.2 Timing, communication and the resulting information structure

To explore how communication influences the information structure I consider the timeline of the contracting game.⁶

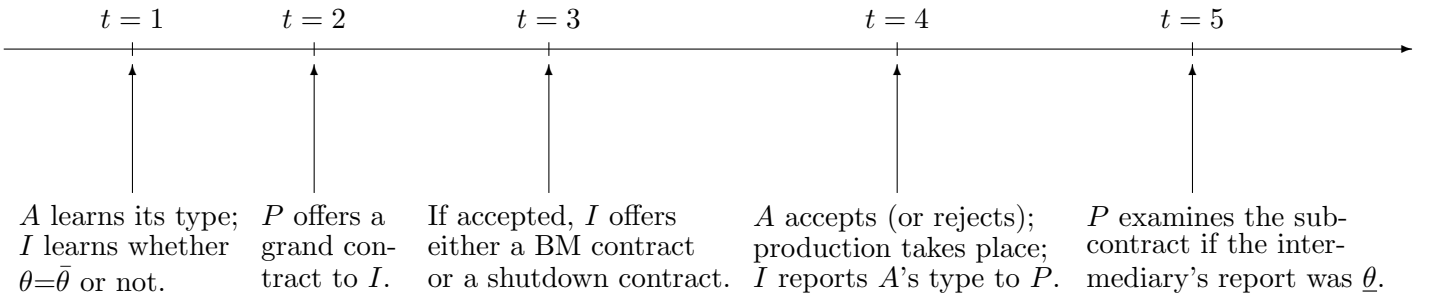


Fig. 1: Timing of the contracting game

⁶The timing and the contracting structure, including the feasibility of communication, remain the same as in FGM, only that I have added a control stage at $t = 5$.

The two following observations help to understand the information structure and the impact of communication.

- The intermediary's report on the agent's *type* is always truthful if a sub-contract was accepted. This follows from the observability of realized outputs together with the (limited) use of budgeting in the setting. Although outputs remain verifiable, the transfers eventually paid to the agent are only partially so, which follows the nature of the BM-contract.
- The intermediary has an informational advantage over the principal, namely to be able to make an unobserved offer at stage $t = 3$. In one state of nature this advantage cannot be limited. This is the case when the intermediary has tried to offer a shutdown contract but the agent was of type $\hat{\theta}$ and has turned down the contract offer. Whenever no subcontract was signed, no hard information can become available.

There is a particular form of internal control that follows from the timing of the contracting game. Since budgeting is now used in the FGM setup, an examination of documents permits a straightforward measure of control. All this comes without loss of generality.

2.3 Rents and grand contract under internal control

We now restate the rents paid to the intermediary. The use of internal control reduces $V(\hat{v})$, and the new incentive constraint reads

$$V(\hat{v}) \geq \nu[\underline{\varrho}V(\hat{v} + \Delta\theta\hat{q} - P^s) + (1 - \underline{\varrho})V(\hat{v} + \Delta\theta\hat{q})]. \quad (5a)$$

The intermediary is now inflicted a penalty P^s with probability $\underline{\varrho}$ if the examination of the written subcontract detects the offer of a shutdown contract, while with probability $1 - \underline{\varrho}$ she keeps her information rent as before. This simple control scheme does not need to involve probabilistic auditing: as long as the principal commands a costly but fully revealing examination technology it is sufficient for the principal to have one state of nature examined, following the intermediary's report.

Because of limited wealth of the agent already expressed in FGM, the assumption of endogenous punishment is straightforward, and P^s is set equal to $\Delta\theta\hat{q}$. This reduces (5a) to

$$V(\hat{v}) \geq \nu[\underline{\varrho}V(\hat{v}) + (1 - \underline{\varrho})V(\hat{v} + \Delta\theta\hat{q})]. \quad (5b)$$

Control costs $c(\varrho)$ are assumed to be strictly convex, with $c(0) = 0$, $c' > 0$, and $c'' > 0$. The Inada conditions $c'(0) = 0$, $c'(1) = +\infty$ are assumed to hold. With (1) binding to prevent the intermediate type from misreporting, (5b) now rewrites into

$$V(\hat{v}) \geq \nu(1 - \underline{\varrho})V(\hat{v} + \Delta\theta\hat{q}). \quad (5c)$$

Solving (5c) for the risk averse intermediary now reduces the intermediary's rent from $\hat{v} = \frac{1}{r} \ln \frac{1 - \nu e^{-r\Delta\theta\hat{q}}}{1 - \nu}$ to $\hat{v} = \frac{1}{r} \ln \frac{1 - \nu(1 - \underline{\varrho})e^{-r\Delta\theta\hat{q}}}{1 - \nu(1 - \underline{\varrho})}$. This expression is decreasing in $\underline{\varrho}$: other things equal, a higher probability of examination reduces the rent that the intermediary can require to give up gambling.

The grand contract, in addition to output targets q and budgets s , now includes the probability of an examination $\varrho(\hat{\theta})$, leading to the following expression for the principal's program in its reduced form:

$$\begin{aligned} \max_{\{(s, q), (\hat{s}, \hat{q}), (\underline{\varrho}), (\bar{s})\}} & (1 - p)[\nu(S(\underline{q}) - \underline{\theta}q - \frac{1}{r} \ln \frac{1 - \nu(1 - \underline{\varrho})e^{-r\Delta\theta\hat{q}}}{1 - \nu(1 - \underline{\varrho})} - \Delta\theta\hat{q} \\ & - c(\underline{\varrho}) + (1 - \nu)(S(\hat{q}) - \hat{\theta}\hat{q} - \frac{1}{r} \ln \frac{1 - \nu(1 - \underline{\varrho})e^{-r\Delta\theta\hat{q}}}{1 - \nu(1 - \underline{\varrho})})] - p\bar{s}, \end{aligned} \quad (6)$$

Proposition 1 *The optimal grand contract under internal control entails:*

- Constraints (1), (2), and (3) are binding. Constraint (5) is replaced by (5c), which is binding as well. All other constraints are strictly satisfied.
- A decreasing schedule of outputs with no distortion for the most efficient type of productive agent:

$$S(\underline{q}^C) = \underline{\theta},$$

and a reduced downward distortion compared to FGM given to the $\hat{\theta}$ -agent, which is explicitly defined by

$$S(\hat{q}^C) = \hat{\theta} + \frac{\nu}{1-\nu} \Delta\theta + \frac{1}{1-\nu} \frac{\nu \Delta\theta (1 - \underline{\varrho}^C) e^{-r\Delta\theta\hat{q}}}{1 - \nu(1 - \underline{\varrho}^C) e^{-r\Delta\theta\hat{q}}}.$$

- An examination of the sub-contract after a report $\tilde{\theta} = \underline{\theta}$.⁷

3 Discussion

3.1 Limits on punishment

While any rent payment to the intermediary in state $\bar{\theta}$ is excluded in this form of contracting, she may still require an upper bound on the penalty to accept the contract. This could make it harder for the principal to exercise internal control. This subsection shows that the findings are robust under limits on punishment. To model such limits in a tractable way, the stake $\Delta\theta\hat{q}$ is multiplied with a factor $\ell \in [0, 1]$, with a lower ℓ implying a higher bound on the penalty and a higher rent payment to the intermediary. Condition (5a) now changes into

$$V(\hat{v}) \geq \nu[\underline{\varrho}V(\hat{v} + (1 - \ell)\Delta\theta\hat{q}) + (1 - \underline{\varrho})V(\hat{v} + \Delta\theta\hat{q})]. \quad (5a \text{ LP})$$

For $\ell = 0$, (5a LP) reduces to (5) in the original FGM setting with no punishment possible. In turn, $\ell = 1$ yields (5a), the setting under full endogenous punishment. If this upper bound on the penalty is required, the principal satisfies (5a LP) by increasing \hat{v} to $-\frac{1}{r} \ln \frac{1-\nu}{1-\nu(\underline{\varrho}e^{-r\Delta\theta\hat{q}} \cdot e^{r\ell\Delta\theta\hat{q}} + (1-\underline{\varrho})e^{-r\Delta\theta\hat{q}})}$.⁸ For any given ℓ , this rent of the intermediary is increasing in ν : in settings where state $\underline{\theta}$ is more likely to occur, the rent of the intermediary is higher.

⁷The internal control scheme presented here assumes that there is no useful information to be detected when no contract exists, which is equivalent to the case when the intermediary reports $\tilde{\theta} = \bar{\theta}$. Otherwise, the principal can extend the scheme toward an examination *both* after a $\bar{\theta}$ and a $\underline{\theta}$ report. This would require to introduce a limited liability constraint different from 0 so that a penalty can also be imposed without production taking place. The probability of examination then changes to $\bar{\varrho}$, and a punishment k is inflicted on the intermediary in the case of a detected lie after a $\bar{\theta}$ report. The principal's program then changes to

$$\begin{aligned} & \max_{\{(s, q), (\hat{s}, \hat{q}), (\bar{\varrho}), (\bar{s})\}} (1-p)[\nu(S(\underline{q}) - \underline{\theta}q - \frac{1}{r} \ln \frac{1-\nu(1-\bar{\varrho})e^{-r\Delta\theta\hat{q}}}{1-\nu(1-\bar{\varrho})} - \Delta\theta\hat{q} \\ & - c(\bar{\varrho}) + (1-\nu)(S(\hat{q}) - \hat{\theta}\hat{q} - \frac{1}{r} \ln \frac{1-\nu(1-\bar{\varrho})e^{-r\Delta\theta\hat{q}}}{1-\nu(1-\bar{\varrho})})] + p(\bar{\varrho}k - c(\bar{\varrho})). \end{aligned} \quad (7)$$

While under this scheme the second-best distortions derived in *Proposition 1* remain unchanged, the principal's option to in addition examine the setting after a $\bar{\theta}$ report improves contracting because of the level effect that the additional punishment now creates in case of a detected lie. Like in *Proposition 1*, the principal always inflicts a punishment in the same state of nature he examines the contract and incurs the costs of examination. I am grateful to an unknown referee for pointing me toward this option. A detailed discussion is available from the author.

⁸See Proof of Proposition 2 for a derivation of this term.

3.2 Comparing delegated contracting with centralization

Analyzing first the intermediary's rent \hat{v} from a comparative statics perspective reveals that both r and $\Delta\theta$ determine the principal's surplus. The more risk averse the intermediary, the easier it is for the principal to satisfy truth-telling: the intermediary's temptation to misbehave is less when she is more risk averse, it vanishes in the limit when r goes to infinity.

Second, $\Delta\theta$, besides being an expression of the intermediary's rent, is also a measure of uncertainty for the principal. Since he has no influence over the size of $\Delta\theta$, it is of interest to examine the uncertainty that he faces when offering the grand-contract. The cross-derivative of the principal's program furthermore reveals that this uncertainty is strictly increasing with the degree of risk aversion, r :

$$\frac{\partial(\cdot)}{\partial\Delta\theta\partial r} = \frac{\varepsilon\nu^2(1-\varrho)^2rq^2e^{-r\Delta\theta\hat{q}}}{(1-\nu-e^{-r\Delta\theta\hat{q}}\nu(1-\varrho))^2}. \quad (8)$$

The derivative itself is decreasing in both $\Delta\theta$ and ϱ .

Third, continuing the comparative statics analysis now permits conclusions on the relative performance of institutions. I assume that the principal has the choice between using delegated contracting with control versus direct contracting that comes at an assumed fixed cost c_d . This is equivalent of arguing that the principal could perform the intermediary's task himself at this cost, which leads to the following reduced-form expression of his program:

$$\max_{\{q,\hat{q}\}} \nu(S(q) - \theta q - \Delta\theta\hat{q}) + (1-\nu)(S(\hat{q}) - \hat{\theta}\hat{q}) - c_d. \quad (9)$$

Comparing this contract with the grand-contract in *Proposition 1* now delivers a criterion of the relative performance of the two schemes, which is summarized in the following proposition.

Proposition 2 *Whenever the principal has a choice between contracting the agent directly at a fixed cost c_d versus delegating the contract offer and using internal control, direct contracting dominates delegated contracting as long as*

$$c_d \leq (1-p) \left(\frac{1}{r} \ln \frac{1-\nu}{1-\nu(\varrho e^{-r\Delta\theta\hat{q}} \cdot e^{r\ell\Delta\theta\hat{q}} + (1-\varrho)e^{-r\Delta\theta\hat{q}})} \right).$$

The expression in the large parenthesis is the negative of the intermediary's rent. As a first result, a lower precision p of the intermediary's possible detection of $\bar{\theta}$ makes delegation less valuable compared to centralization. In turn, a higher probability ν of the efficient state $\underline{\theta}$ increases both the intermediary's rent and decreases the relative performance of delegated contracting.

This particular result sheds light on the impact the different legal frameworks have on the principal's ability to impose penalties. The higher the limits on punishments required (the lower ℓ), the more the setting converges to the original FGM model, making a higher distortion necessary at the optimum, and reducing the relative efficiency of delegated contracting versus centralization. In turn, low or no limits on penalties (e.g. following public law regulating the control of civil servants) may render the extended setting on delegated contracting relatively superior.

4 Conclusion

Delegated contracting involves more than restoring delegation-proofness through increasing the rent payment to an intermediary. The same setting permits the principal to make use of what at first seems to be a central drawback of this contractual form, namely its particular information flow. This paper shows that the principal

can extend the grand contract toward an internal form of control that typically applies to vertical hierarchies. The results are robust.

Furthermore, the paper has compared internal control under delegated contracting with direct contracting. Internal control renders delegated contracting more efficient. This illustrates why delegated hierarchies typically use forms of internal control to limit the discretion of intermediate players. Internal control has an impact, even if there is no communication possible between the top and bottom tier of the hierarchy.

Comparisons with other contractual settings that permit a richer treatment of communication are left for future research.

5 Acknowledgments

I am grateful to Philippe Aghion, Fahad Khalil, Dongsoo Shin, Jean Tirole, the editor, and an unknown referee for important suggestions that helped to improve this paper. All remaining errors are mine.

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7 Appendix

Proof of Proposition 1. Deriving (6) w.r. to outputs leads to the requested result.

For any $\varrho^C > 0$, this expression is strictly less than the R.H.S. in the FGM solution, which in the here adopted notation condenses to $\hat{\theta} + \frac{\nu}{1-\nu} \Delta\theta + \frac{1}{1-\nu} \frac{\nu\Delta\theta}{e^{r\Delta\theta\hat{a}} - \nu}$.

Internal control involves an examination of the *existing* subcontract after the intermediary's report of $\tilde{\theta} = \underline{\theta}$. This reduces the distortion of the output target offered to the $\hat{\theta}$ -type and improves contracting. The discussion of the three possible reports reveals:

Case 1: Assume the intermediary reports $\tilde{\theta} = \bar{\theta}$. No sub-contract exists and the report can either be true false. If the report was true, the intermediary did exactly her job to *not* offer a sub-contract when observing that $\theta = \bar{\theta}$.

If the report was false, the intermediary has offered a shutdown contract and lost her gamble. Although she did not act following the principal's intention, she was unable to reap any rent. The agent's true type is then $\theta = \hat{\theta}$.

Case 2: The intermediary reports $\tilde{\theta} = \hat{\theta}$. This report must be true: the sub-contract was accepted, which reveals that the intermediary has indeed offered a BM-style contract designed for both types of firms, otherwise the revealed $\hat{\theta}$ -type agent would have rejected. The form of budgeting used in FGM is sufficient to reveal information in this state.

Case 3: The intermediary reports $\tilde{\theta} = \underline{\theta}$. Only this case permits the use of hard information accessible to the principal through managerial control, without accessing the productive agent. The accepted sub-contract can be a shutdown contract, in which case the intermediary may have reaped the virtual costs $\Delta\theta\hat{q}$ included in the budget to permit a BM contract. Iff the agent is of type $\underline{\theta}$, examination of the existing contract reveals that a shutdown contract was offered. Other than under auditing of a productive agent where the principal audits the inefficient state and inflicts a punishment when the agent was found to be efficient, the principal examines the contract when type $\underline{\theta}$ is announced, plus inflicts a punishment when the real type was indeed of type $\underline{\theta}$ but the intermediary offered a shutdown contract. ■

Proof of Proposition 2.

It is easy to see that the principal's costs of direct contracting need to be less or equal the cost of the hierarchy under internal control, covering the intermediary's rent to reach delegation-proofness. The term in the large parenthesis is found by modifying (5a) into (5a LP):

$$V(\hat{v}) \geq \nu[\underline{\wp}V(\hat{v} + \Delta\theta\hat{q} - (\ell \cdot \Delta\theta\hat{q})) + (1 - \underline{\wp})V(\hat{v} + \Delta\theta\hat{q})].$$

Solving for \hat{v} is done by using $V(x) = \frac{1-e^{-rx}}{r}$ and isolating the resulting $e^{-r\hat{v}}$ terms on both sides. Rearranging the terms leads to the expression in the proposition. ■